Interest Rate Derivatives: The Standard Market Models

Chapter 28

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The Complications in Valuing Interest Rate Derivatives (page 647)

- We need a whole term structure to define the level of interest rates at any time
- The stochastic process for an interest rate is more complicated than that for a stock price
- Volatilities of different points on the term structure are different
- Interest rates are used for discounting the payoff as well as for defining the payoff

Approaches to Pricing Interest Rate Options

- Use a variant of Black's model
- Use a no-arbitrage (yield curve based) model

Black's Model

- Similar to the model proposed by Fischer Black for valuing options on futures
- Assumes that the value of an interest rate, a bond price, or some other variable at a particular time T in the future has a lognormal distribution

Black's Model for European Bond Options (Equations 28.1 and 28.2, page 648)

- Assume that the future bond price is lognormal

$$c = P(0,T)[F_B N(d_1) - K N(d_2)]$$

$$p = P(0,T)[K N(-d_2) - F_B N(-d_1)]$$

$$d_1 = \frac{\ln(F_B / K) + \sigma_B^2 T / 2}{\sigma_B \sqrt{T}}; \quad d_2 = d_1 - \sigma_B \sqrt{T}$$

- Both the bond price and the strike price should be cash prices not quoted prices

Forward Bond and Forward Yield

Approximate duration relation between forward bond price, F_B, and forward bond yield, y_F

$$\frac{\Delta F_B}{F_B} \approx - D \Delta y_F \text{ or } \frac{\Delta F_B}{F_B} \approx - D y_F \frac{\Delta y_F}{y_F}$$

where D is the (modified) duration of the forward bond at option maturity

Yield Vols vs Price Vols
(Equation 28.4, page 651)

- This relationship implies the following approximation

$$\sigma_B = D y_0 \sigma_y$$

where σ_y is the forward yield volatility, σ_B is the forward price volatility, and y_0 is today's forward yield

- Often σ_y is quoted with the understanding that this relationship will be used to calculate σ_B

Theoretical Justification for Bond Option Model

Working in a world that is FRN wrt a zero-coupon bond maturing at time T, the option price is

$$P(0,T)E_T[\max(B_T - K, 0)]$$

Also

$$E_T[B_T] = F_B$$

This leads to Black's model

Caps and Floors

- A cap is a portfolio of call options on LIBOR. It has the effect of guaranteeing that the interest rate in each of a number of future periods will not rise above a certain level
- Payoff at time t_{k+1} is $L\delta_k \max(R_k\text{-}R_K, 0)$ where L is the principal, $\delta_k = t_{k+1}\text{-}t_k$, R_K is the cap rate, and R_k is the rate at time t_k for the period between t_k and t_{k+1}
- A floor is similarly a portfolio of put options on LIBOR. Payoff at time t_{k+1} is $L\delta_k \max(R_K - R_k, 0)$

Caplets

- A cap is a portfolio of “caplets”
- Each caplet is a call option on a future LIBOR rate with the payoff occurring in arrears
- When using Black’s model we assume that the interest rate underlying each caplet is lognormal

Black's Model for Caps (p. 657)

- The value of a caplet, for period (t_k, t_{k+1}) is

$$L\delta_k P(0,t_{k+1})[F_k N(d_1) - R_K N(d_2)]$$

$$\text{where } d_1 = \frac{\ln(F_k/R_K) + \sigma_k^2 t_k/2}{\sigma_k\sqrt{t_k}} \text{ and } d_2 = d_1 - \sigma\sqrt{t_k}$$

- F_k : forward interest rate for (t_k, t_{k+1})
- σ_k : forward rate volatility
- L: principal
- R_K : cap rate
- $\delta_k = t_{k+1} - t_k$

When Applying Black's Model To Caps We Must ...

- EITHER
  - Use spot volatilities
  - Volatility different for each caplet
- OR
  - Use flat volatilities
  - Volatility same for each caplet within a particular cap but varies according to life of cap

Theoretical Justification for Cap Model

Working in a world that is FRN wrt a zero-coupon bond maturing at time t_{k+1} the option price is

$$P(0,t_{k+1})E_{k+1}[\max(R_k - R_K, 0)]$$

Also

$$E_{k+1}[R_k] = F_k$$

This leads to Black's model

Swaptions

- A swaption or swap option gives the holder the right to enter into an interest rate swap in the future
- Two kinds
  - The right to pay a specified fixed rate and receive LIBOR
  - The right to receive a specified fixed rate and pay LIBOR

Black's Model for European Swaptions

- When valuing European swap options it is usual to assume that the swap rate is lognormal
- Consider a swaption which gives the right to pay s_K on an n-year swap starting at time T. The payoff on each swap payment date is

$$\frac{L}{m}\max(s_T - s_K, 0)$$

where L is principal, m is payment frequency and s_T is market swap rate at time T

Black's Model for European Swaptions continued (Equation 28.11, page 659)

The value of the swaption is

$$LA[s_0 N(d_1) - s_K N(d_2)]$$

where $d_1 = \dfrac{\ln(s_0/s_K) + \sigma^2 T/2}{\sigma\sqrt{T}}$; $d_2 = d_1 - \sigma\sqrt{T}$

s_0 is the forward swap rate; σ is the forward swap rate volatility; t_i is the time from today until the i th swap payment and

$$A = \frac{1}{m}\sum_{i=1}^{mn} P(0, t_i)$$

Theoretical Justification for Swap Option Model

Working in a world that is FRN wrt the annuity underlying the swap, the option price is

$$LAE_A[\max(s_T - s_K, 0)]$$

Also

$$E_A[s_T] = s_0$$

This leads to Black's model

Relationship Between Swaptions and Bond Options

- An interest rate swap can be regarded as the exchange of a fixed-rate bond for a floating-rate bond
- A swaption or swap option is therefore an option to exchange a fixed-rate bond for a floating-rate bond

Relationship Between Swaptions and Bond Options (continued)

- At the start of the swap the floating-rate bond is worth par so that the swaption can be viewed as an option to exchange a fixed-rate bond for par
- An option on a swap where fixed is paid and floating is received is a put option on the bond with a strike price of par
- When floating is paid and fixed is received, it is a call option on the bond with a strike price of par

Deltas of Interest Rate Derivatives

Alternatives:

- Calculate a DV01 (the impact of a 1bps parallel shift in the zero curve)
- Calculate impact of small change in the quote for each instrument used to calculate the zero curve
- Divide zero curve (or forward curve) into buckets and calculate the impact of a shift in each bucket
- Carry out a principal components analysis for changes in the zero curve. Calculate delta with respect to each of the first two or three factors